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# New DA white dwarf evolutionary models and their pulsational properties

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**Abstract.** In this letter we investigate the pulsational properties of ZZ Ceti stars on the basis of new white dwarf evolutionary models calculated in a self-consistent way with the predictions of time dependent element diffusion and nuclear burning. In addition, full account is taken of the evolutionary stages prior to the white dwarf formation. Emphasis is placed on the trapping properties of such models. By means of adiabatic, non-radial pulsation calculations, we find, as a result of time dependent diffusion, a much weaker mode trapping effect, particularly for the high-period regime of the pulsation *g*-spectrum. This result is valid at least for models with massive hydrogen-rich envelopes. Thus, mode trapping would not be an effective mechanism to explain the fact that all the high periods expected from standard models of stratified white dwarfs are not observed in the ZZ Ceti stars.

**Key words.** stars: evolution – stars: interiors – stars: white dwarfs – stars: oscillations

## 1. Introduction

Pulsating DA white dwarfs (WD) or ZZ Ceti stars have captured the attention of numerous researchers since the first star (HL Tau 76, Landolt 1968) belonging to this class was reported to exhibit multi-periodic luminosity variations (McGraw 1979). Over the last two decades, various studies have presented strong evidence that pulsating DA WDs represent an evolutionary stage in the cooling history of the majority, if not all, DA WDs. Rapid progress in the study of these pulsating stars has been possible thanks to the development of powerful theoretical tools paralleled by an increasing degree of sophistication in observational techniques. A major step towards the understanding of ZZ Ceti pulsations was given by Dolez & Vauclair (1981) and Winget et al. (1982) who independently demonstrated that models of ZZ Ceti stars have pulsationally unstable *g*-modes due to the  $\kappa - \gamma$  mechanism. From then on, the asteroseismology of DA WDs has provided invaluable in-

sights on their internal structure and evolution. (Tassoul et al. 1990; Brassard et al. 1991, 1992ab; Gautschy et al. 1996 and Bradley 1996, 1998, 2001 amongst others).

An important aspect of pulsating WDs is related to the trapping properties. Mode trapping in compositionally stratified WDs has been invoked to explain the longstanding fact that all the modes expected from theoretical models are not actually observed in the ZZ Ceti stars (Winget et al. 1981; Brassard et al. 1992a). In this scenario, certain modes are characterized by local wavelengths that are comparable to the thickness of one of the compositional layer, particularly the hydrogen-rich envelope. The importance of trapped modes lies on the fact that they appear to be the most likely to be observed because they require low kinetic energies to reach observable amplitudes. More specifically, the amplitude of the eigenfunctions of modes trapped in the hydrogen envelope is small in the core, which causes such modes to have low oscillation kinetic energy as compared with adjacent modes. This behaviour manifests itself as local minima in kinetic energy versus period diagrams. In particular, trapped modes characterized by periods close to the thermal time-scale of the driving region will reach high enough amplitudes for them to be observed. This picture has been reinforced by non-adiabatic calculations (Dolez & Vauclair 1981; Winget et al. 1982). However, recent evidence seems to cast some doubts on the correlation between observed amplitudes

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and mode trapping. Indeed, recent seismological studies of ZZ Ceti stars (e.g. Bradley 1998) point to the fact that the observed periods having the largest amplitudes in the power spectrum do not correspond to trapped modes as predicted by the best fitting model.

The pulsation properties depend on the details of the WD modeling. This is particularly true regarding the abundance distribution at the chemical interfaces, mostly at the hydrogen-helium transition. In this connection, most of the existing calculations invoke diffusive equilibrium in the trace element approximation to assess the shape of the hydrogen-helium transition (Tassoul et al. 1990; Brassard et al. 1992ab; Bradley 1996). However, equilibrium conditions may not be achieved at the base of massive hydrogen envelopes, even at the characteristic ages of ZZ Ceti stars (see Iben & MacDonald 1985). In view of these concerns, we have recently carried out new evolutionary calculations for DA WD stars which take fully into account time dependent element diffusion, nuclear burning and the history of the WD progenitor in a self-consistent way. The present letter is aimed at specifically exploring the trapping properties of such models.

## 2. Evolutionary models

The WD models on which the present results are based have been calculated by means of a detailed evolutionary code developed by us at La Plata Observatory. The code has been employed in previous studies on WD evolution (Althaus et al. 2001ab) and it has recently been modified to study the evolutionary stages prior to the WD formation (see Althaus et al. 2001c). The constitutive physics include: up-to-date OPAL radiative opacities for different metallicities, conductive opacities, neutrino emission rates, a detailed equation of state and a complete network of thermonuclear reaction rates for hydrogen and helium burning (see Althaus et al. 2001c). For a proper treatment of the diffusively evolving chemical stratification, gravitational settling and the thermal and chemical diffusion of nuclear species have been considered.

The evolutionary stages prior to the WD formation have been fully taken into account. Specifically, we started our calculations from a  $3 M_{\odot}$  star at the zero-age main sequence and we follow its further evolution all the way from the stage of hydrogen and helium burning in the core up to the tip of the asymptotic giant branch where helium thermal pulses occur. After experiencing 11 thermal pulses, the model is forced to evolve towards its WD configuration by invoking strong mass loss episodes. As a result, a WD remnant of  $0.563 M_{\odot}$  is obtained. The evolution of this remnant is pursued through the stage of planetary nebulae nucleus to the domain of the ZZ Ceti stars on the WD cooling branch. An important aspect of these calculations is related to the evolution of the chemical abundance during the WD cooling. In particular, the shape of the composition transition zones is of the utmost importance regarding the pulsational properties of the ZZ Ceti models. In this respect, diffusion processes cause near

discontinuities in the abundance distribution at the start of the cooling branch to be considerably smoothed out by the time the ZZ Ceti domain is reached. This can be appreciated in Fig. 1, which also illustrates the profile of the hydrogen-helium interface resulting from the predictions of diffusive equilibrium in the trace element approximation (thin dotted line). The shape of the innermost carbon and oxygen distribution emerges from the chemical rehomogenization process due to the Rayleigh-Taylor instability occurring at early stages of the WD evolution (see Althaus et al. 2001c and also Salaris et al. 1997)<sup>1</sup>. Surrounding the carbon-oxygen interior there is a shell rich in both carbon ( $\approx 35\%$ ) and helium ( $\approx 60\%$ ), and a overlying layer consisting of nearly pure helium of mass  $0.003 M_{\odot}$ . The presence of carbon in the helium-rich region below the helium buffer stems from the short-lived convective mixing episode that has driven the carbon-rich zone upwards during the peak of the last helium pulse on the asymptotic giant branch. We want to mention that the total helium content within the star once helium shell burning is eventually extinguished amounts to  $0.014 M_{\odot}$  and that the mass of hydrogen that is left at the start of the cooling branch is about  $1.5 \times 10^{-4} M_{\odot}$ , which is reduced to  $7 \times 10^{-5} M_{\odot}$  due to the interplay of residual nuclear burning and element diffusion by the time the ZZ Ceti domain is reached.

## 3. Results

Next, we shall discuss the pulsational properties of a selected WD model at  $T_{\text{eff}} \sim 12000$  K. We should remark that, although the chemical profiles evolve as the WD cools down through the instability strip (see Althaus et al. 2001c), the conclusions of the present paper remain valid for any model belonging to the ZZ Ceti instability strip. We begin by showing in Fig. 2 the square of the Brunt-Väisälä frequency  $N$  (computed as in Brassard et al. 1991) and the Ledoux term  $B$  of such a model. The results for the diffusive equilibrium approximation are also plotted as thin lines. Note the smooth shape of  $B$ , which is a direct consequence of the chemical abundance distribution. The contributions from the Ledoux term are characterized by extended tails, and translate into smooth bumps on  $N^2$ .

The characteristic of  $B$  and  $N^2$  as predicted by our models is markedly different from those found in previous studies in which the WD evolution is treated in a simplified way, particularly regarding the chemical abundance distribution (e.g. Tassoul et al. 1990; Brassard et al. 1991, 1992ab; Bradley 1996). Clearly, non-equilibrium chemical profiles lead to  $B$  values with markedly less pronounced peaks as compared with the diffusive equilibrium treatment. Accordingly, the Brunt-Väisälä frequency turns out to be smoother as a result of non-equilibrium diffusion.

<sup>1</sup> Before rehomogenization, the shape of the carbon and oxygen profile towards the centre is characterized by an off-centered peak typical of evolutionary calculations in which semi-convection and overshooting are not considered (see Mazzitelli & D'Antona 1986).

For the pulsation analysis we have employed the general Newton-Raphson code described in Córscico & Benvenuto (2001). We have computed g-modes with  $\ell = 1, 2$  and  $3$  with periods in the range of  $50 \text{ s} \lesssim P_k \lesssim 1300 \text{ s}$  ( $k$  being the radial order of modes). The upper panels of Figs. 3, 4 and 5 show, respectively, the values of oscillation kinetic energy for modes with  $\ell = 1, 2$  and  $3$  in terms of computed periods. Lower panels depict the corresponding values for the forward period spacing  $\Delta P_k$  ( $\equiv P_{k+1} - P_k$ ). Filled dots depict the results corresponding to our model with non-equilibrium diffusion, whereas empty dots indicate the results predicted by the diffusive equilibrium approximation for the hydrogen-helium interface. In the interests of clarity, the scale for the kinetic energy in the case of diffusive equilibrium is displaced upwards by 1 dex.

For the non-equilibrium diffusion model the quantities plotted (especially the  $E_{\text{kin}}$  values) exhibit two clearly different trends. Indeed, for  $P_k \gtrsim 600 \text{ s}$  and irrespective of the value of  $\ell$ , the kinetic energy of adjacent modes is quite similar, which is in contrast with the situation found for lower periods. Interestingly, the  $\Delta P_k$  minima are commonly associated with  $E_{\text{kin}}$  maxima, but that modes with  $E_{\text{kin}}$  maxima are adjacent to the modes with  $E_{\text{kin}}$  minima. On the other hand, the period spacing diagrams show appreciable variations of  $\Delta P_k$  for  $P_k \lesssim 600 \text{ s}$ . This is due mostly to the presence of chemical abundance transitions in DA WD models as explained by Brassard et al (1992ab). In contrast, for higher periods the  $\Delta P_k$  of the modes tend to a constant, asymptotic value (Tassoul 1980).

The assumption of diffusive equilibrium in the trace element approximation in WD modeling gives rise to a kinetic energy spectrum and period spacing distribution in which the presence of the well known mode trapping phenomenon is clearly visible, as previously reported by numerous investigators (see Brassard et al. 1992b, particularly their figures 20a and 21a for the case of  $M_{\text{H}} = 10^{-4} M_{\ast}$ ). The trapped modes correspond to modes with local minima in  $\log(E_{\text{kin}})$  and  $\Delta P_k$ . Here, we find that these trapping properties virtually vanish when account is made of WD models with diffusively evolving stratifications. This is particularly true for large periods, though for low periods trapping is also substantially affected. We attribute the differences found between both treatments to the markedly different shapes of the Ledoux term at the hydrogen-helium interface as predicted by non- and equilibrium diffusion.

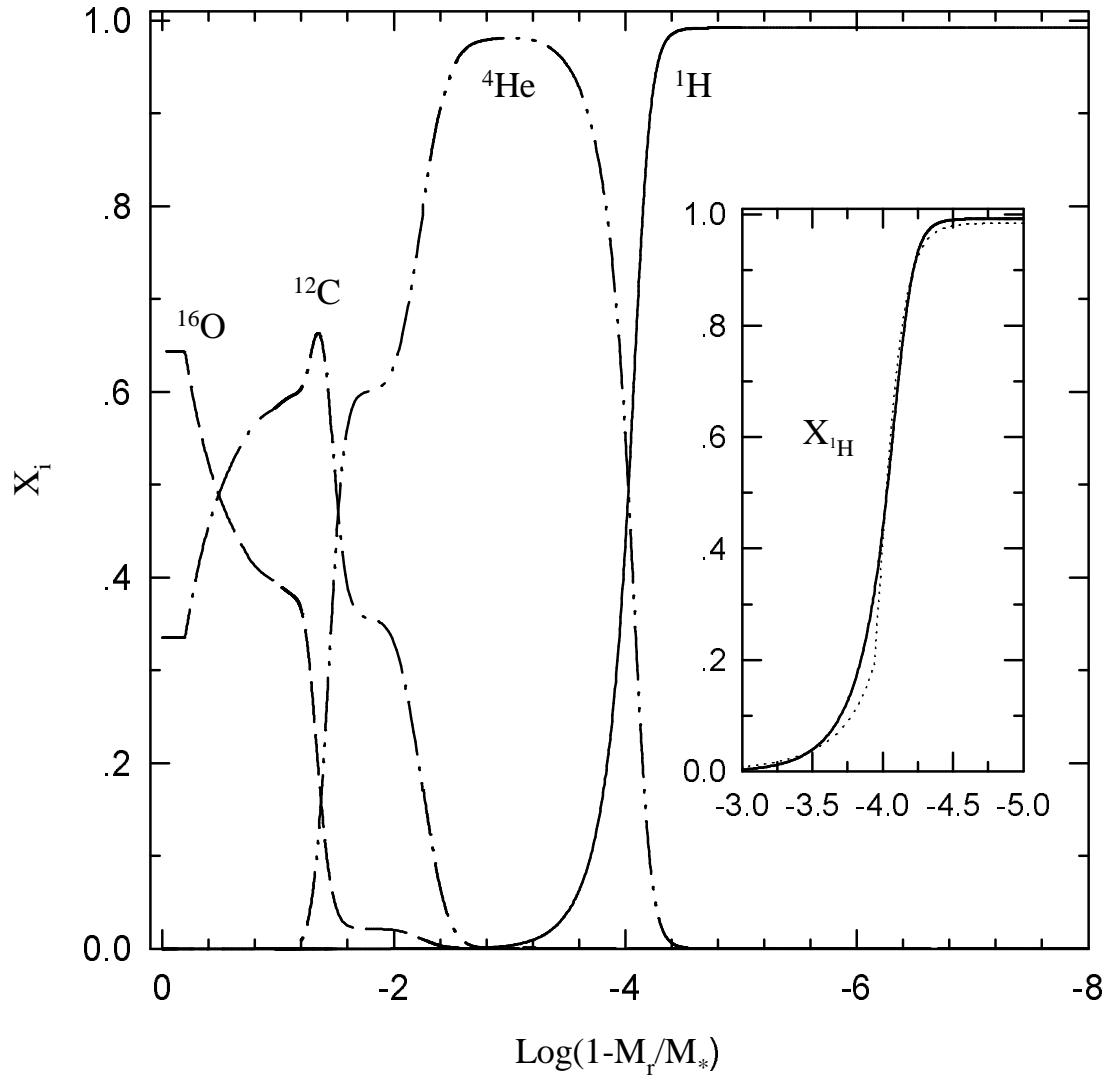
From the results presented in this letter we judge that, for high periods, trapping mechanism in massive envelopes of stratified WDs is not an appropriate one to explain the fact that all the modes expected from theoretical models are not observed in ZZ Ceti stars. It is worth mentioning that Gautschy & Althaus (2001) have recently found, on the basis of a consistent diffusion modeling, a weaker trapping effect on the periodicities in DB WDs. Our results give strong theoretical support to recent evidence against the claimed correlation between the observed luminosity variations amplitude and trapping of

modes. Finally, to place these assertions on a firmer basis, a non-adiabatic stability analysis of the pulsational properties of non-equilibrium diffusion models is required. A more extensive exploration of the results presented in this letter will be presented in a future work.

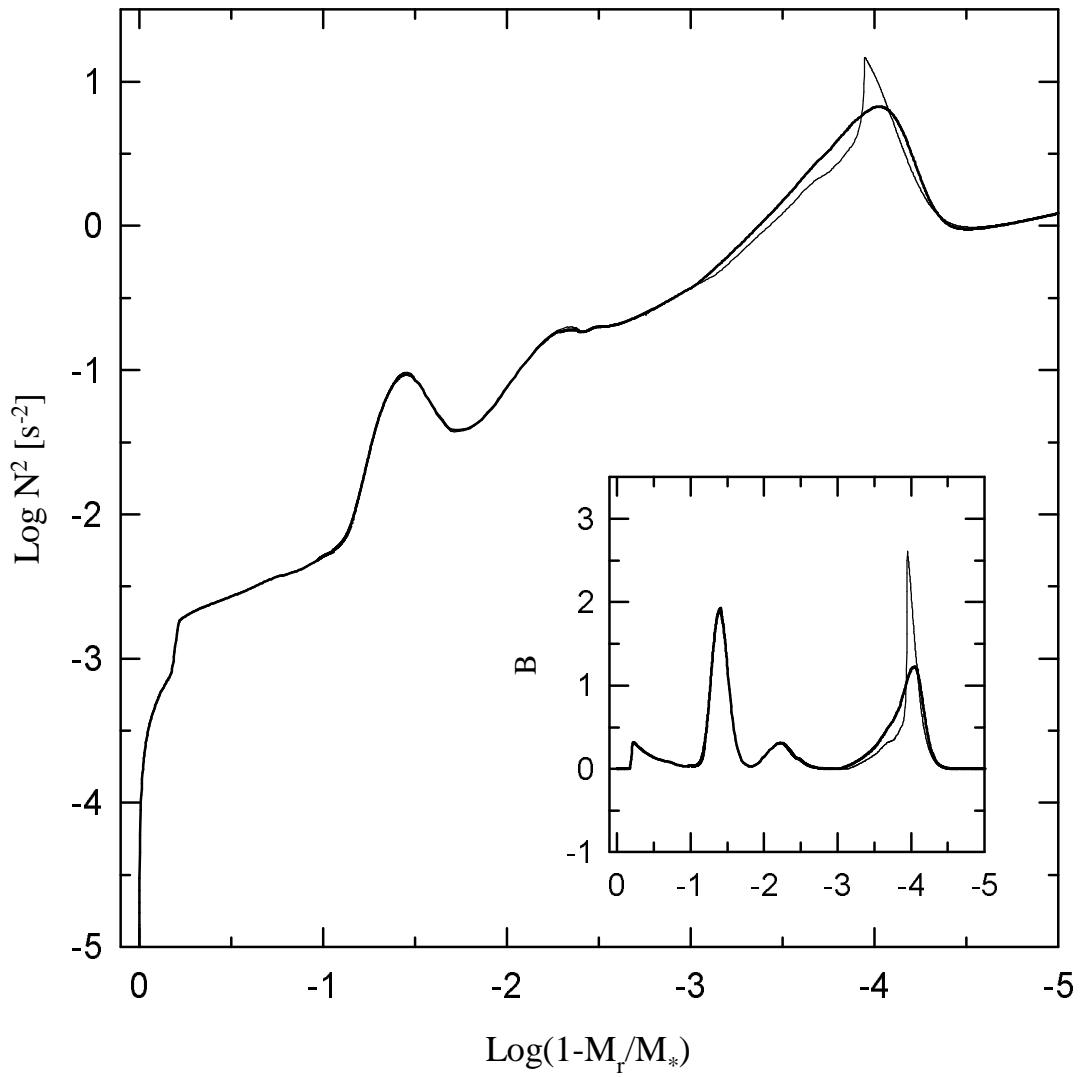
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## References

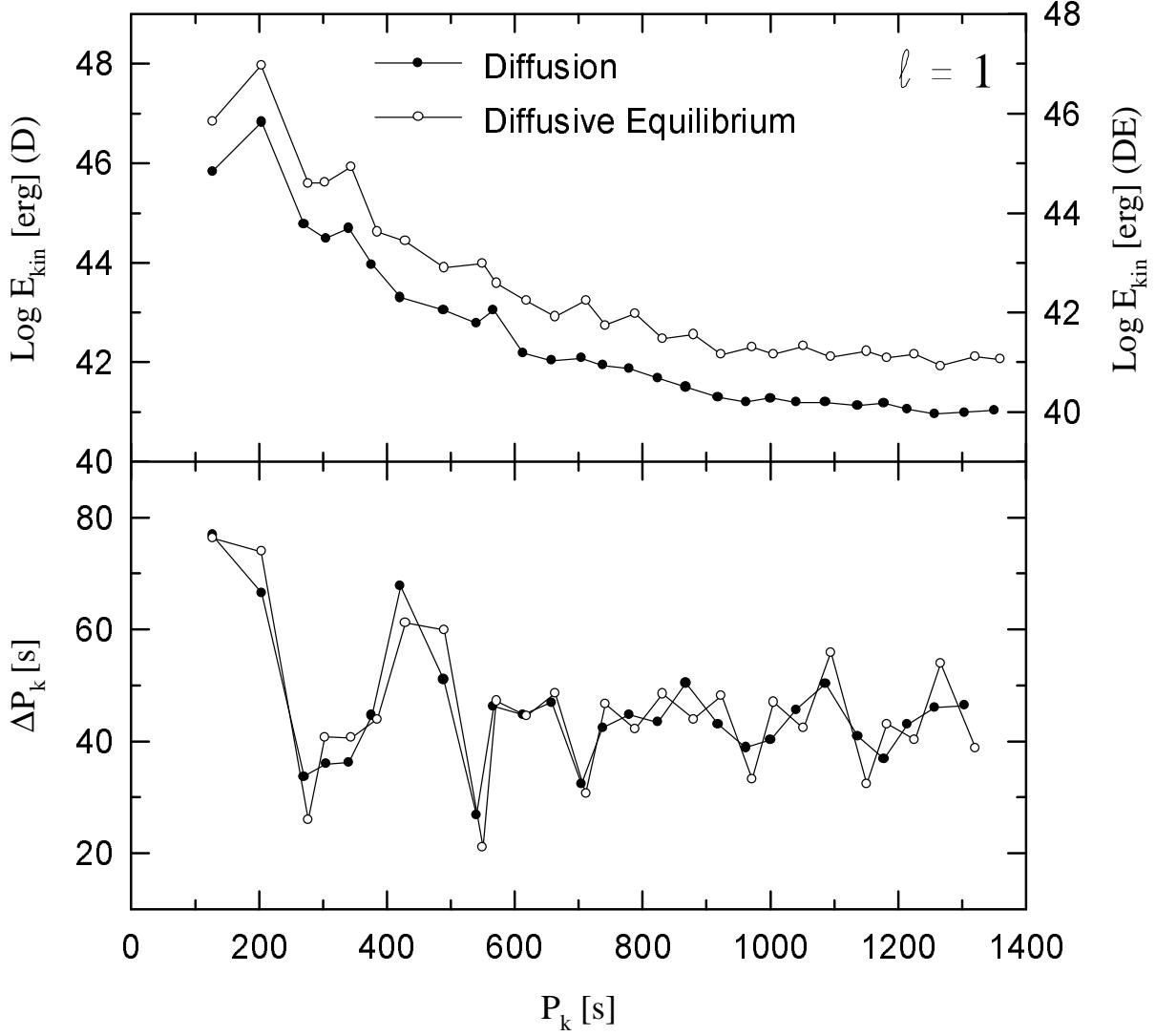
- Althaus L. G., Serenelli A. M., Benvenuto O. G., 2001a, ApJ, 554, 1110
- Althaus L. G., Serenelli A. M., Benvenuto O. G., 2001b, MNRAS, 323, 471
- Althaus L. G., Serenelli A. M., Córscico A. H., Benvenuto O. G., 2001c, MNRAS, submitted
- Bradley P. A., 1996, ApJ, 468, 350
- Bradley P. A., 1998, ApJS, 116, 307
- Bradley P. A., 2001, ApJ, 552, 326
- Brassard P., Fontaine G., Wesemael F., Hansen C. J., 1992a, ApJS, 80, 369
- Brassard P., Fontaine G., Wesemael F., Tassoul M., 1992b, ApJS, 81, 747
- Brassard P., Fontaine G., Wesemael F., Kawaler S. D., Tassoul M., 1991, ApJ, 367, 601
- Córscico A. H., Benvenuto O. G., 2001, Ap&SS, to be published (astro-ph: 0104267)
- Dolez N., Vauclair G., 1981, A&A, 102, 375
- Gautschy A., Althaus L. G., 2001, A&A, submitted
- Iben I. Jr., MacDonald J., 1985, ApJ, 296, 540
- Landolt A. U., 1968, ApJ, 153, 161
- Mazzitelli I., D'Antona F., 1986, ApJ, 308, 706
- McGraw J. T., 1979, ApJ, 229, 203
- Salaris M., Domínguez I., García-Berro E., Hernanz M., Isern J., Mochkovitch R., 1997, ApJ, 486, 413
- Tassoul M., 1980, ApJS, 43, 469
- Tassoul M., Fontaine G., Winget D. E., 1990, ApJS, 72, 335
- Winget D. E., Van Horn H. M., Hansen C. J., 1981, ApJ, 245, L33
- Winget D. E., Van Horn H. M., Tassoul M., Hansen C. J., Fontaine G., Carroll B. W., 1982, ApJ, 252, L65



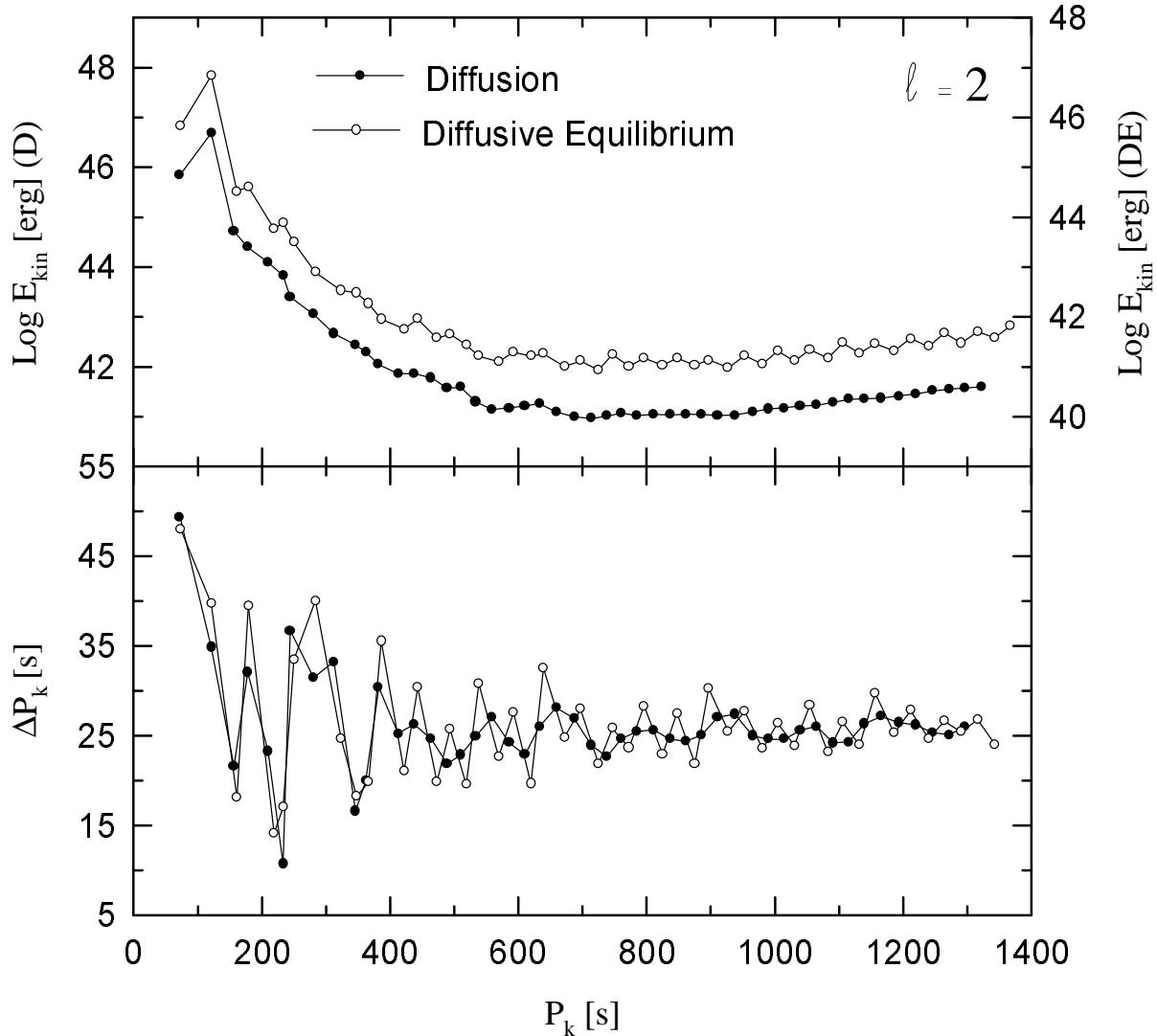
**Fig. 1.** Internal chemical profiles for hydrogen, helium, carbon and oxygen. The hydrogen-helium interface resulting from the predictions of non-equilibrium diffusion (diffusive equilibrium) are shown with solid line (thin dotted line) in the inset. The WD stellar mass is  $0.563 M_{\odot}$  and the effective temperature is  $\approx 12000\text{K}$ .



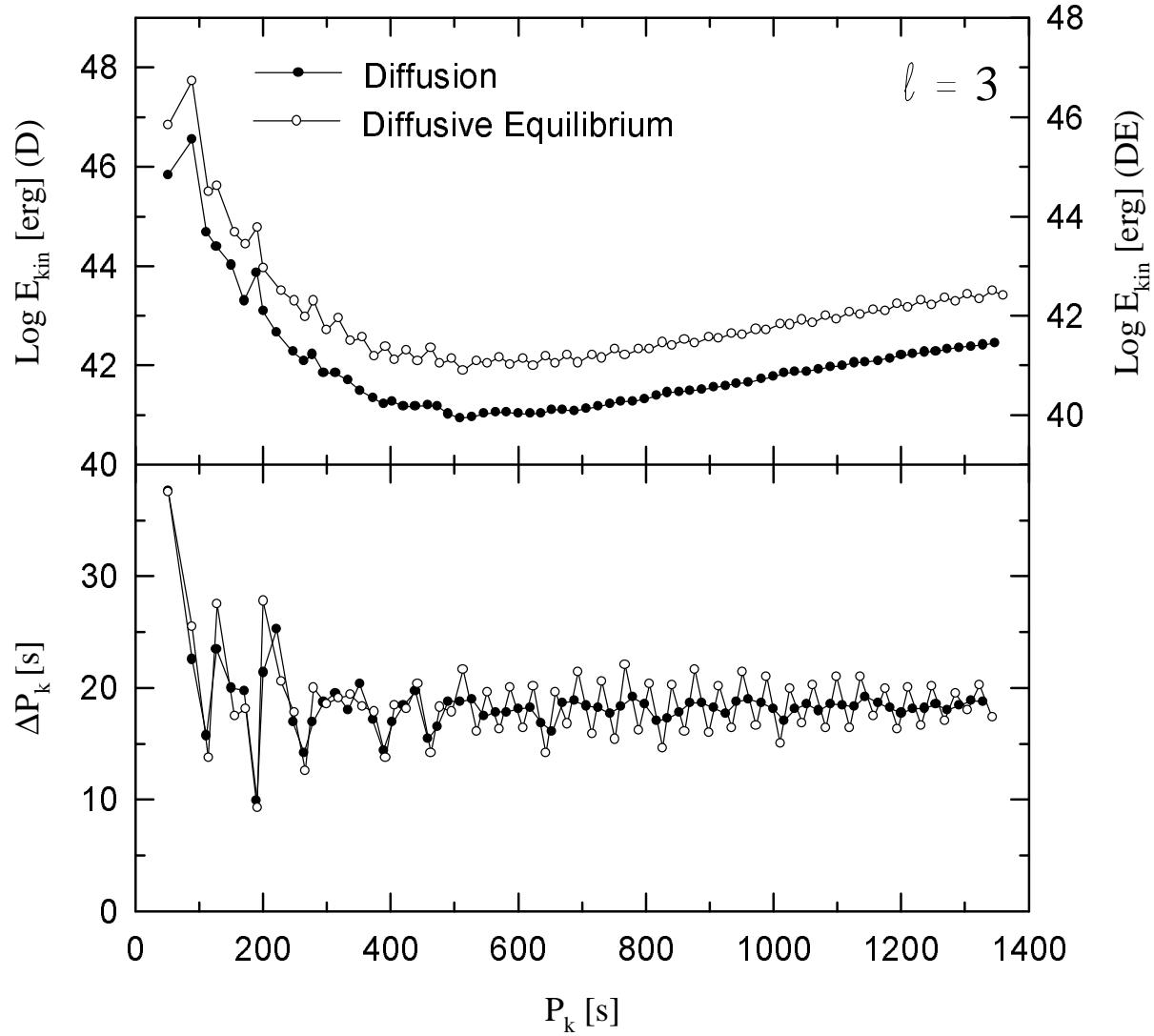
**Fig. 2.** The logarithm of the squared Brunt-Väisälä frequency and the Ledoux term,  $B$ , for the non-equilibrium diffusion model. The results for the diffusive equilibrium approximation are shown in thin lines.



**Fig. 3.** Oscillation kinetic energy (upper panel) and period spacing (lower panel) values for  $\ell = 1$  in terms of the computed periods,  $P_k$ . Filled dots correspond to pulsational computations for the non-equilibrium diffusion model, and empty dots for the diffusive equilibrium one. In the interests of clarity, the scale for the kinetic energy in the case of diffusive equilibrium is displaced upwards by 1 dex. The kinetic energy values correspond to the normalization  $y_1 = \delta r/r = 1$  at the surface of the non-perturbed models for each mode.



**Fig. 4.** Same as Fig. 3 but for  $\ell = 2$ .



**Fig. 5.** Same as Fig. 3 but for  $\ell = 3$ .